Shapes

CS 491 – Competitive Programming

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Objectives

- Know some of the basic formulae for
  - Circles
  - Triangles
  - Squares
- Most code samples from Competitive Programming 3.

Testing if Inside

- For a circle, you need a center (a, b) and a radius r.
- All points $(x-a)^2 + (y-b)^2 = r^2$

```
int insideCircle(point_i p, point_i c, int r) { // all inte
    int dx = p.x - c.x, dy = p.y - c.y;
    int Euc = dx * dx + dy * dy, rSq = r * r;
     // all integer
    return Euc < rSq ? 0 : Euc == rSq ? 1 : 2;
    // 0 = inside, 1 = on border, 2 = outside
}
```

Basic Formulae

- Value of π is `acos(-1.0)`
- Diameter $d = 2r$
- Circumference is $2\pi r$, area is πr^2
- Given two points `p1` and `p2` and radius `r`, we can compute the circles:

```
bool circle2PtsRad(point p1, point p2, double r, point &c)
    double d2 = (p1.x - p2.x) * (p1.x - p2.x) +
                (p1.y - p2.y) * (p1.y - p2.y);
    double det = r * r / d2 - 0.25;
    if (det < 0.0) return false;
    double h = sqrt(det);
    c.x = (p1.x + p2.x) * 0.5 + (p1.y - p2.y) * h;
    c.y = (p1.y + p2.y) * 0.5 + (p2.x - p1.x) * h;
    return true;
}
```

Types

- Types of triangles:

Equilateral	All three sides the same, all angles are 60 degrees
Isosceles	Two edges the same, two degrees the same.
Scalene	All edges different
Right	One angle 90 degrees

Area Calculations

Given	sides a, b, c
Perimeter	$p = a + b + c, s = \frac{p}{2}$
Area	$\sqrt{s(s-a)(s-b)(s-c)}$

Circles in Triangles

- A triangle with area A and semi-perimeter s has an inscribed circle (incircle) with radius $r = A/s$.

```
double rInCircle(double ab, double bc, double ca) {
    return area(ab, bc, ca) / (0.5 * perimeter(ab, bc, ca)); }

double rInCircle(point a, point b, point c) {
    return rInCircle(dist(a, b), dist(b, c), dist(c, a)); }
```

Center of Inscribed Circle

```
int inCircle(point p1, point p2, point p3,
             point &ctr, double &r) {
  r = rInCircle(p1, p2, p3);
  if (fabs(r) < EPS) return 0; // no inCircle center
  line l1, l2; // compute these two angle bisectors
  double ratio = dist(p1, p2) / dist(p1, p3);
  point p = translate(p2, scale(toVec(p2, p3),
                               ratio / (1 + ratio)));
  pointsToLine(p1, p, l1);
  ratio = dist(p2, p1) / dist(p2, p3);
  p = translate(p1, scale(toVec(p1, p3),
                         ratio / (1 + ratio)));
  pointsToLine(p2, p, l2);
  areIntersect(l1, l2, ctr);
  return 1; }
```

Circumscribed Circles

- Triangle can be enclosed by an *circumscribed cirlce*:
- Radius is $R = a * b * c/(4 * area(a, b, c))$

```
double rCircumCircle(double ab, double bc, double ca) {
    return ab * bc * ca / (4.0 * area(ab, bc, ca)); }
double rCircumCircle(point a, point b, point c) {
    return rCircumCircle(dist(a, b), dist(b, c), dist(c, a))
```

Trigonometrics

Is a triangle possible? $a + b > c$, where c is largest side.

Pythagorean Theorem $a^2 + b^2 = c^2$

Law of Sines $\frac{a}{sin(\alpha)} = \frac{b}{sin(\beta)} = \frac{c}{sin(\gamma)} = 2R$

Law of Cosines $c^2 = a^2 + b^2 - 2 \times a \times b \times cos(\gamma)$